B.K. BIRLA CENTRE FOR EDUCATION

SARALA BIRLA GROUP OF SCHOOLS
A CBSE DAY-CUM-BOYS' RESIDENTIAL SCHOOL

NABET

INDIAN PUBLIC SCHOOLS' CONFERENCE

TERM-1 EXAMINATION, 2025-26
MARKING SCHEME – APPLIED MATHEMATICS (241)

Class: XII	Time: 3 hrs
Date: 05/09/25	Max Marks: 80
Adm No:	Roll.No.

General Instructions:
1. This Question Paper has 5 Sections A, B, C, D and E.
2. Section A has 20 MCQs carrying 1 mark each
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case-based integrated units of assessment (04 marks each) with sub-parts.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks have been provided. An internal choice has been provided in the 2marks questions of Section E
8. Draw neat figures wherever required. Take π =22/7 wherever required if not stated.

SECTION A

1.	If $x \equiv 4 \bmod (7)$, then positive values of x are				1m
	(a) { 4,11,18,…}	(b) { 11,18,25…}	(c) { 4,8,12,…}	(d) none of these	
2.	In a 300 metre race A beats B by 22.5 metre or six seconds .B's time over the race course is				1m
	(a) 80 sec	(b) 82 sec	(c) 76 sec	(d) none of these	
3.	If $\lvert x-2\rvert \geq 7, x \in R$, then				1m
	(a)$x \in [-5,9]$	(b) $x \in (-5,9]$	*(c) $x \in (-\infty, -5] \cup [9, \infty)$*	(d) none of these	
4.	If $x \in R$, $\lvert x\rvert \leq 9$, then				1m
	(a) $-9 \leq x \leq 9$	(b) $x \geq 9$	(c) $x \leq -9$	(d) none of these	
5.	The probability of guessing at least 8 correct answers out of 10 true-false questions is				1m
	(a) 7/64	*(b) 7/128*	(c) 7/256	(d) no solutions	
6.	For a binomial variate X, if n = 4 and P(X = 0) = 16/81 , then P(X = 4) is				1m
	(a) 1/3	(b) 1/27	(c) *1/81*	(d) none of these	
7.	The number of all possible matrices of order 3 x 3 with entry 0 or 1 is				1m
	(a) 18	(b) 27	(c) 81	*(d) 512*	
8.	If A and B are symmetric matrices of same order , then AB – BA is a				1m
	(a) Symmetric matrix	*(b) Skew symmetric*	(c) Zero matrix	(d) none of these	
9.	If A is a square matrix of order 3 and | A | = 2 , then the value of | -AA'| is				1m
	(a) 4	(b) 2	(c) -2	*(d) none of these*	
10.	If A is a square matrix of order 3 X 3 such that | A | = 4 , then | 3A | is equal to				1m
	(a) 27	(b) 81	*(c) 108*	(d) none of these	
11.	If $x = at^2$, $y = 2at$, then y" =				1m
	(a) $-1/2at^3$	(b) $-1/2at^2$	(c) $1/t^2$	(d) none of these	

12. **Derivative of log x w.r.t 1/x is** **1m**

(a) $-1/x^3$ **(b) $-1/x$** *(c) $-x$* **(d) none of these**

13 **If the marginal revenue function of a commodity is MR = $2x - 9x^2$, then the revenue function is** **1m**

(a) $2x^2 - 9x^3$ **(b) $2 - 18x$** *(c) $x^2 - 3x^3$* **(d) none of these**

14 **If the demand function for a commodity is p = $20 - 2x - x^2$ and the market demand is 3 units then consumer's surplus is** **1m**

(a) 27 **(b) 38** **(c) 42** **(d) 47**

15 **What is the value of the definite integral** **1m**

$\int_1^2 (x^2+1)dx$?

(a) 7/3 **(b) 10/3** **(c) 4** ***(d) none of these***

16 **The demand function for a commodity is given by P=100−2x. The supply function is S=10+x. What is the consumer surplus at the market equilibrium point?** **1m**

(a) 300 *(b) 900* **(c) 600** **(d) none of these**

17 **What is the general solution of the differential equation** **1m**

$\frac{dy}{dx} = \frac{y}{x}$**?**

(a) y = x/c *(b) y = cx* **(c) y = c + x** **(d) none of these**

18 **Type II error occurs when:** **1m**

(a) The null hypothesis is false, but we fail to reject it.	**(b) The null hypothesis is true, but we reject it.**	**(c) The null hypothesis is false, and we reject it.**	**(d) none of these**

19 **Assertion (A):** **1m**

In a Poisson distribution, the mean and variance are equal.

Reason (R):

The Poisson distribution is used to model events that occur randomly and independently over a fixed interval of time or space

(a) Both Assertion (A) and Reason (R) are the true and Reason (R) is a correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not a correct explanation of Assertion (A).

(c) Assertion (A) is true and Reason (R) is false.

(d) Assertion (A) is false and Reason (R) is true.

20 **Assertion (A): The solution to the inequality $5x+10 \leq 0$ is the set of all real numbers such that $x \geq -2$.** **1m**

Reason (R): To isolate x, you must subtract 10 from both sides and then divide by 5.

(a) Both Assertion (A) and Reason (R) are the true and Reason (R) is a correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not a correct explanation of Assertion (A).

(c) Assertion (A) is true and Reason (R) is false.

(d) Assertion (A) is false and Reason (R) is true.

SECTION B

21 **Find y', if $y^x + x^y + +x^x = a^b$.** **2m**

A:-
1. Write equation: $y^x + x^y + x^x = ab$ (RHS constant ⇒ derivative 0).
2. Use formula $\frac{d}{dx}u^v = u^v(v' \ln u + v\frac{u'}{u})$.
3. Differentiate each term:
   - $\frac{d}{dx}y^x = y^x(x\frac{y'}{y} + \ln y)$,
   - $\frac{d}{dx}x^y = x^y(y' \ln x + \frac{y}{x})$,
   - $\frac{d}{dx}x^x = x^x(\ln x + 1)$.
4. Sum and set = 0: **1m**

$$y^x\left(x\frac{y'}{y} + \ln y\right) + x^y\left(y' \ln x + \frac{y}{x}\right) + x^x(\ln x + 1) = 0.$$

5. Collect y' terms:

$$y'\left(y^x\frac{x}{y} + x^y \ln x\right) + \left(y^x \ln y + x^y\frac{y}{x} + x^x(\ln x + 1)\right) = 0.$$

6. Solve for y':

$$\boxed{y' = -\frac{y^x \ln y + x^y\frac{y}{x} + x^x(\ln x + 1)}{y^x\frac{x}{y} + x^y \ln x}} \quad (\text{assume } x > 0, y > 0 \text{ and denominator} \neq 0).$$ **1m**

OR

Find y' when
$x^y + y^x = \log a$

A:-
1. Differentiate both sides ($\log a$ is constant ⇒ derivative 0).
   Use $\frac{d}{dx}u^v = u^v(v' \ln u + v\frac{u'}{u})$.
2. $\frac{d}{dx}x^y = x^y(y' \ln x + \frac{y}{x})$
   $\frac{d}{dx}y^x = y^x(x\frac{y'}{y} + \ln y)$
3. Equation after differentiation: **1m**

$$x^y\left(y' \ln x + \frac{y}{x}\right) + y^x\left(x\frac{y'}{y} + \ln y\right) = 0$$

4. Group y' terms:

$$y'(x^y \ln x + y^x\frac{x}{y}) + (x^y\frac{y}{x} + y^x \ln y) = 0$$

5. Solve for y':

$$\boxed{y' = -\frac{x^y\frac{y}{x} + y^x \ln y}{x^y \ln x + y^x\frac{x}{y}}}$$ **1m**

22 **Can you find the values of x and y so that the matrices** **2m**
$\begin{bmatrix} 3x+7 & 5 \\ y+1 & 2-3x \end{bmatrix}$ *and* $\begin{bmatrix} 0 & y-2 \\ 8 & 4 \end{bmatrix}$ **may be equal?**

A: - **x = -7/3** **1m**
y=-2/3 **1m**

23 **Using Cramer's rule , solve the following system of linear equations:** **2m**
(a + b)x – (a – b)y = 4ab
(a – b)x + (a + b)y = 2(a^2 – b^2)

A:- **x = a+b** **1m**
y = a – b **1m**

24 **In what ratio must a grocer mix two varieties of pulses costing Rs. 15 per kg and Rs. 20 per kg respectively so as to get a mixture worth Rs 16.50 per kg?** **2m**

OR

A can run a kilometre in 4 minutes 54 sec. and B in 5 min. How many metres start can A give B in a kim race. So that the race may and in a dead heat ?

A:- **Ratio of mixing pulses**
Cost of cheaper = Rs. 15/kg
Cost of dearer = Rs. 20/kg
Mean price = Rs. 16.50/kg **1m**
Ratio = 1.5 : 3.5 = 3 : 7
Answer: Mix in the ratio 3 : 7 (cheaper : dearer). **1m**

Or

A's time for 1 km = 4 min 54 sec = 294 sec **1m**
B's time for 1 km = 300 sec
Start = 1000 – 980 = 20 m
Answer: A can give B a 20 m start. **1m**

25 **Find the order and degree of the differential equation:** **2m**

$$\left(\frac{d^2y}{dx^2}\right)^3 + \frac{dy}{dx} = \sin x$$

A:-
- **Order** = 2 (highest order derivative is $\frac{d^2y}{dx^2}$) **1m**
- **Degree** = 3 (power of the highest order derivative after removing radicals and fractions). **1m**

Answer: Order = 2, Degree = 3 ✅

SECTION C

26 **Find all pair of consecutive even positive integers, both of which are larger than 5, such that their sum is less than 23** **3m**

OR

Solve the following system of linear inequalities: 4x-5 <11 and **−3𝑥 −4 ≥ 8 .**

A:- **Let the number be x**
x>5 and x + x + 2 < 23 **2m**
5 < x< 10.5 **1m**

Or

4x – 5 < 11, -3x-4 >=8
x < 4 , x<= -4 **2m**
x belongs to (-ꝏ,-4) **1m**

27 **If x = (e^t + e^{-t})/2 and y = (e^t - e^{-t})/2 , show y^{2}y" +xy'-y = 0** **3m**

A:-
1. $x = \frac{e^t+e^{-t}}{2},\ y = \frac{e^t-e^{-t}}{2}$
   $\Rightarrow \frac{dx}{dt} = y,\ \frac{dy}{dt} = x$
2. $y' = \frac{dy}{dx} = \frac{x}{y}$
3. $\frac{d}{dt}(y') = \frac{y^2-x^2}{y^2} = -\frac{1}{y^2}$ (since $x^2 - y^2 = 1$) **2m**
4. $y'' = \frac{-1/y^2}{y} = -\frac{1}{y^3}$
5. $y^2y'' + xy' - y = -\frac{1}{y} + \frac{x^2}{y} - y = \frac{-1+x^2-y^2}{y} = 0$

1m

OR

Find second order derivative of log(logx).

A:- Let $y = \log(\log x)$ (natural logs).

First derivative:

$$y' = \frac{1}{\log x} \cdot \frac{1}{x} = \frac{1}{x \log x}.$$ 1m

Second derivative:

Differentiate $\frac{1}{x \log x}$:

$$y'' = -\frac{1 + \log x}{x^2 (\log x)^2}.$$

$$\boxed{y'' = -\frac{1 + \log x}{x^2 (\log x)^2}}$$ 2m

28 Express the following matrix as the sum of a symmetric matrix and a skew symmetric matrix and verify the result: **3m**

$$\begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$$

A:- Calculations up to (A + A')/2 **1m**

………(A-A')/2…….. **1m**

Verification **1m**

29 Find the maximum value of (log x)/x , x > 0. **3m**

A:- Step 1: Differentiate

$$f'(x) = \frac{\frac{1}{x} \cdot x - \log x \cdot 1}{x^2} = \frac{1 - \log x}{x^2}$$

Step 2: Set derivative to zero

$$1 - \log x = 0 \quad \Rightarrow \quad \log x = 1$$ **2m**

$$x = e$$

Step 3: Maximum value

$$f(e) = \frac{\log e}{e} = \frac{1}{e}$$ **1m**

30 The average height of a random sample of 400 people from a city is 1.75 m. It is known that the population standard deviation is 40 **3m**

(a) Determine the 90% confidence interval for the population mean.

(b) Determine the 95% confidence interval for the population mean.

A:- Standard error:

$$SE = \frac{\sigma}{\sqrt{n}} = \frac{0.40}{\sqrt{400}} = \frac{0.40}{20} = 0.02$$

(a) 90% C.I.

$z_{0.05} = 1.645$

Margin of error = $1.645 \times 0.02 = 0.0329$

$$\text{C.I.} = (1.75 - 0.0329,\ 1.75 + 0.0329) = (1.7171,\ 1.7829)$$ **2m**

(b) 95% C.I.

$z_{0.025} = 1.96$

Margin of error = $1.96 \times 0.02 = 0.0392$

$$\text{C.I.} = (1.7108,\ 1.7892)$$ **1m**

OR

A sample of 100 Maruti authorised service centres showed 13 are in Delhi, 18 in Mumbai, 17 in Chennai and 15 in Kolkata.

(i) Develop an estimate of the proportion of Maruti Service centres in Delhi.

(ii) Develop an estimate of the proportion of Maruti Service centres in Chennai.
(iii) Develop an estimate of the proportion of Maruti Service centres that are not in these four cities.

A:- Given: $n = 100$

(i) **Proportion in Delhi**

$$\hat{p}_{Delhi} = \frac{13}{100} = 0.13$$ **1m**

(ii) **Proportion in Chennai**

$$\hat{p}_{Chennai} = \frac{17}{100} = 0.17$$ **1m**

(iii) **Proportion not in the four cities**

Total in four cities = $13 + 18 + 17 + 15 = 63$

$$\hat{p}_{Others} = \frac{100 - 63}{100} = \frac{37}{100} = 0.37$$ **1m**

31 **A river near a small-town floods and overflows twice in every 10 years on an average. Assuming that the Poisson distribution is appropriate, what is the mean expectation? Also, calculate the probability of 3 or less overflows and floods in a 10-year interval.** **3m**
[Given $e^{-2} = 0.13534$}

A:- To Find:

1. **Mean expectation** = $\boxed{2}$
2. $P(X \leq 3) = P(0) + P(1) + P(2) + P(3)$

Use Poisson formula:

$$P(x) = \frac{e^{-2} \cdot 2^x}{x!}$$

- $P(0) = 0.13534$
- $P(1) = 0.13534 \times 2 = 0.27068$ **2m**
- $P(2) = 0.27068$
- $P(3) = 0.13534 \times \frac{8}{6} = 0.18045$

$$P(X \leq 3) = 0.13534 + 0.2706 \downarrow 0.27068 + 0.18045 = \boxed{0.85715}$$ **1m**

SECTION D

32 **Evaluate the following** **5m**

(i) $\int \frac{dx}{2x^2+4x-3}$

(ii) $\int \frac{dx}{\sqrt{3x^2+2x-1}}$

A:-

(i) $\int \frac{dx}{2x^2 + 4x - 3} = \frac{1}{2\sqrt{10}} \ln \left| \frac{2x + 2 - \sqrt{10}}{2x + 2 + \sqrt{10}} \right| + C.$ **2m**

(ii) $\int \frac{dx}{\sqrt{3x^2 + 2x - 1}} = \frac{1}{\sqrt{3}} \ln \left| 3x + 1 + \sqrt{3}\sqrt{3x^2 + 2x - 1} \right| + C.$ **3m**

OR

Evaluate the following

(i) $\int \frac{2x+3}{x^2+3x+2}\,dx$

(ii) $\int \frac{4x^2+7x+5}{x(x+1)^2}\,dx$

A:- (i)

$x^2+3x+2=(x+1)(x+2)$

$$\frac{2x+3}{(x+1)(x+2)}=\frac{1}{x+1}+\frac{1}{x+2}$$

$$\boxed{\ln|x+1|+\ln|x+2|+C}$$

2m

(ii)

$$\frac{4x^2+7x+5}{x(x+1)^2}=\frac{A}{x}+\frac{B}{x+1}+\frac{C}{(x+1)^2}$$

Multiply out:

$4x^2+7x+5=A(x+1)^2+Bx(x+1)+Cx$

Comparing coefficients:

$A=5,\ B=-1,\ C=2$

So:

$$\int \frac{5}{x}\,dx-\int \frac{1}{x+1}\,dx+\int \frac{2}{(x+1)^2}\,dx$$

$$\boxed{5\ln|x|-\ln|x+1|-\frac{2}{x+1}+C}$$

3m

33 Using matrix method, solve the following system of equations: **5m**

x – 2y = 10

2x + y + 3z = 8

-2y + z = 7.

A:- Calculations up to adj A **3m**

x = 4, y = -3, z = 1 **2m**

OR

Using Cramer's Rule, solve the following system of equations:

$2x-y+3z=9$

$x+2y-z=8$

$3x-y+2z=10$

A:-

$$\Delta = \begin{vmatrix} 2 & -1 & 3 \\ 1 & 2 & -1 \\ 3 & -1 & 2 \end{vmatrix} = -6$$

$$\Delta_x = \begin{vmatrix} 9 & -1 & 3 \\ 8 & 2 & -1 \\ 10 & -1 & 2 \end{vmatrix} = -13$$

$$\Delta_y = \begin{vmatrix} 2 & 9 & 3 \\ 1 & 8 & -1 \\ 3 & 10 & 2 \end{vmatrix} = -35$$ **3m**

$$\Delta_z = \begin{vmatrix} 2 & -1 & 9 \\ 1 & 2 & 8 \\ 3 & -1 & 10 \end{vmatrix} = 7$$

$$x = \frac{-13}{-6} = \frac{13}{6}, \quad y = \frac{-35}{-6} = \frac{35}{6}, \quad z = \frac{7}{-6} = -\frac{7}{6}$$ **2m**

34 **Find the particular solution of the differential equation $x(1+y^2)\,dx - y(1+x^2)\,dy = 0$ given that y = 1 when x = 0.** **5m**

A:-

Step 1: Rearranging the Equation

Bring all terms to one side:

$$x(1+y^2)\,dx = y(1+x^2)\,dy$$

Separate variables:

$$\frac{x}{1+x^2}\,dx = \frac{y}{1+y^2}\,dy$$ **2m**

Step 2: Integrate Both Sides

$$\int \frac{x}{1+x^2}\,dx = \int \frac{y}{1+y^2}\,dy$$

Use substitution or standard integrals:

$$\frac{1}{2}\ln(1+x^2) = \frac{1}{2}\ln(1+y^2) + C$$

Multiply both sides by 2:

$$\ln(1+x^2) = \ln(1+y^2) + 2C$$ **5m**

35 **The probability of a shooter hitting a target is ¾. How many minimum number of times must he fire so that the probability of hitting the target at least once is more than 0.99 ?** **5m**

A:-

- Probability of hitting = $\frac{3}{4}$
- Probability of missing = $\frac{1}{4}$

We need:

$$1 - \left(\frac{1}{4}\right)^n > 0.99 \Rightarrow \left(\frac{1}{4}\right)^n < 0.01$$ **2m**

Try values of n:

- $\left(\frac{1}{4}\right)^3 = \frac{1}{64} \approx 0.0156$
- $\left(\frac{1}{4}\right)^4 = \frac{1}{256} \approx 0.0039 < 0.01$

✅ **Minimum** $n = \boxed{4}$

3m

SECTION E

36 **Susy is rowing a boat. She takes 6 hours to row 48 km upstream whereas she takes 3 hours to go the same distance downstream.** **4m**

Based on the above situation, answer the following questions:
(a) What is her speed of rowing in still water?
(b) What is the speed of the stream?
(c) What is her average speed?

Or

The stream is flowing at the speed of 4km/h. If Susy rows a certain distance upstream in 3.5 hours and returns to the same place in 1.5 hours, then find the speed of boat?

A:- **(a) Calculations** **1m**
x = 12 km/hr
(b) Speed of stream = 4km/hr **1m**
(c) Average speed = 96/9 km/h **2m**

Or

Speed of boat = 10km/hr

37 **Three students Ram, Mohan and Ankit go to a shop to buy stationary. Ram purchases 2 dozen notebooks, 1 dozen pens and 4 pencils, Mohan purchases 1 dozen notebook, 6 pens and 8 pencils and Ankit purchases 6 notebooks, 4 pens and 6 pencils. A notebook costs ₹15, a pen costs ₹4.50 and a pencil costs ₹1.50. Let *A* and *B* be the matrices representing the number of items purchased by the three students and the prices of the items respectively. Based on the above information, answer the following questions:** **4m**
(a) Find the order of matrix B representing the price of three items
(b) Find the order of matrix A representing items purchased by three students
(c) Find the bill amount of Ram

Or

Find the total bill amount of all three students.

A:- **(a) 3 x 1** **1m**
(b) 3 x 3 **1m**
(c) Rs. 420
Or **2m**
Rs.756

38 **A cable network provider in a small town has 500 used to collect ₹ 300 per month from each subscriber. He proposes to increase the monthly charges and it is believed from past experienced that for every increase of ₹1, one subscriber will discontinue the service.** **4m**
Based on the above information, answer the following questions:
(a) If ₹*x* is the monthly increase in subscription amount, then find the number subscribers left.
(b) Find the total revenue ' *R* ' (in ₹).
(c) Find the number of subscribers which gives the maximum revenue.

Or

Find the maximum revenue generated.

A:-
(a) Subscribers left = $500 - x$ **1m**
(b) $R = (500 - x)(300 + x) = 150000 + 200x - x^2$ **1m**
(c) Max at $x = 100$, subscribers = 400, max revenue = ₹1,60,000 **2m**

*****BEST OF LUCK*****